COMPUTING REACTION TIMES AND PATHS WITH MACHINE LEARNING AND RARE EVENT SAMPLING METHODS

THOMAS PIGEON

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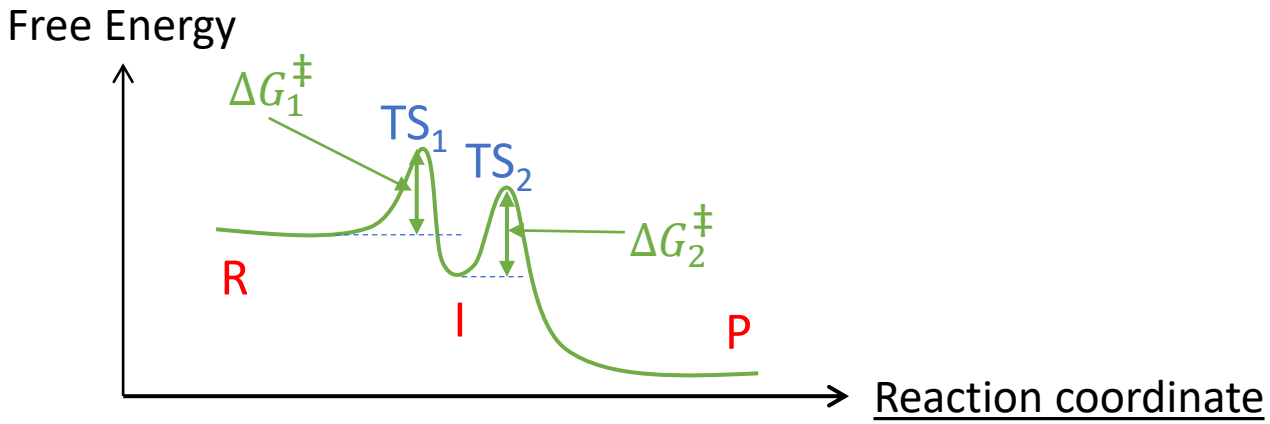
INTRODUCTION: REACTION MECHANISMS

Targets : compute reaction rates

identify reaction mechanism



Different methods exist:

- Transition State Theory (TST): for instance, Eyring-Polanyi equation[1] $\boldsymbol{k}^{hTST} = \frac{k_B T}{h} e^{-\frac{\Delta G^{\ddagger}}{k_B T}}$

  Using free energy computed by static approach within harmonic approximation or Molecular Dynamics (MD)
- **Alternatively: MD and Rare events simulation methods to access directly the reaction time**

  Hill relation[2]: $\boldsymbol{k}^{Hill} = p_{R \to P} \phi_R$

[1] Eyring, H. (1935). The activated complex in chemical reactions. *The Journal of Chemical Physics*, *3*(2), 107-115.

[2] Hill, T. (2012) Free energy transduction in biology: The steady-state kinetic and thermodynamic formalism. *Elsevier Science and Technology Books*

INTRODUCTION: STANDARD MOLECULAR DYNAMICS

Simulates the dynamic of the system by adding a thermostat to newton equations of motion

ex. Langevin formalism[1]

NVE ensemble NVT ensemble

$$\begin{cases} dq_t = M^{-1} p_t dt \\ dp_t = -\nabla V(q_t) dt - \gamma p_t dt + \sqrt{2\gamma M k_B T} dW_t \end{cases}$$

Preserves energy — Dissipate energy — Provides energy

Newton equation — Langevin part

Not efficient for the simulation of rare events due to high energy barriers and entropic bottlenecks

Time scales: integration time step : $\sim 10^{-15} s$ rare event rate $\sim 10^{-9} s^{-1}$ to $10^{3} s^{-1}$

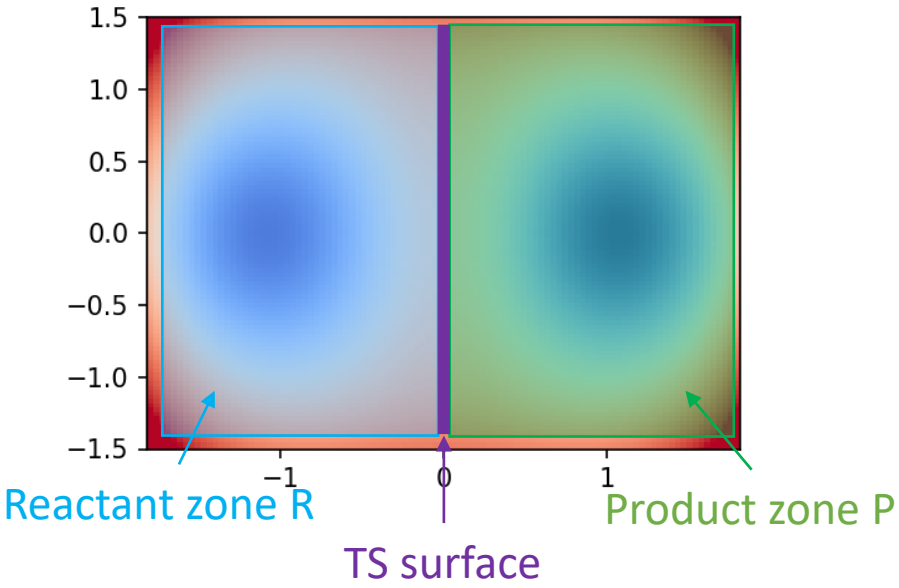
MD based approaches to overcome barriers:

- TST → biased MD such as Metadynamics[2], Blue-Moon sampling[3] …
  Dynamics is lost but rates are estimated from free energy
- Hill → rare events sampling methods such as Adaptive multi-level splitting[4]
  Dynamics preserved thus the rates can directly be computed

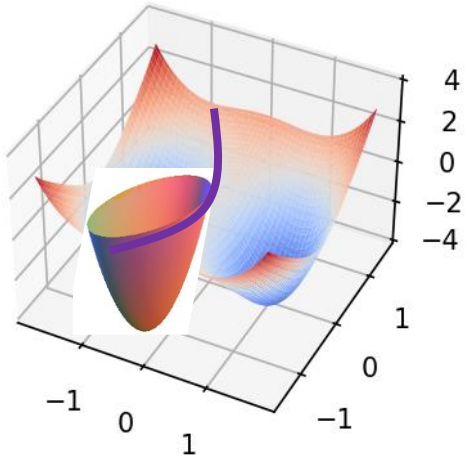
[1] Langevin P. (1908), *Comptes-Rendus de l'Académie des Sciences*, *146*, 530-532
[2] Laio, A., & Parrinello, M. (2002) *Proceedings of the National Academy of Sciences*, *99*(20), 12562-12566.
[3] Carter, E. A., Ciccotti, G., Hynes, J. T., & Kapral, R. (1989). *Chemical Physics Letters*, *156*(5), 472-477.
[4] Cérou, F., & Guyader, A. (2007) *Stochastic Analysis and Applications*, *25*(2), 417-443.

I. ADAPTIVE MULTI-LEVEL SPLITTING METHOD FOR REACTION RATES

Transition State Theory



2-dimensional potential



Rate = probability of being in TS with respect to R × frequency of decomposition to P

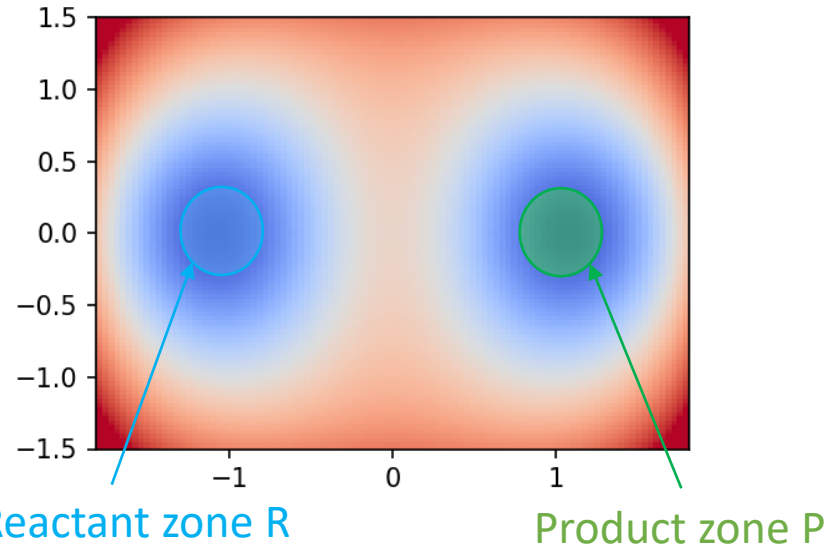
$$\boldsymbol{k}^{TST} = p(TS \mid R)\ \phi_{TS \to P} \qquad \boldsymbol{k}^{hTST} = e^{-\frac{\Delta G^{\ddagger}}{k_B T}} \frac{k_B T}{h}$$

Sensitive to the TS definition

TST overestimates rates (κ)

hTST poorly captures entropy

Hill relation



Rate = probability of reaching P before R starting from ∂R × frequency of exits of R

$$\boldsymbol{k}^{Hill} = p_{R \to P}(\partial R)\ \phi_R$$

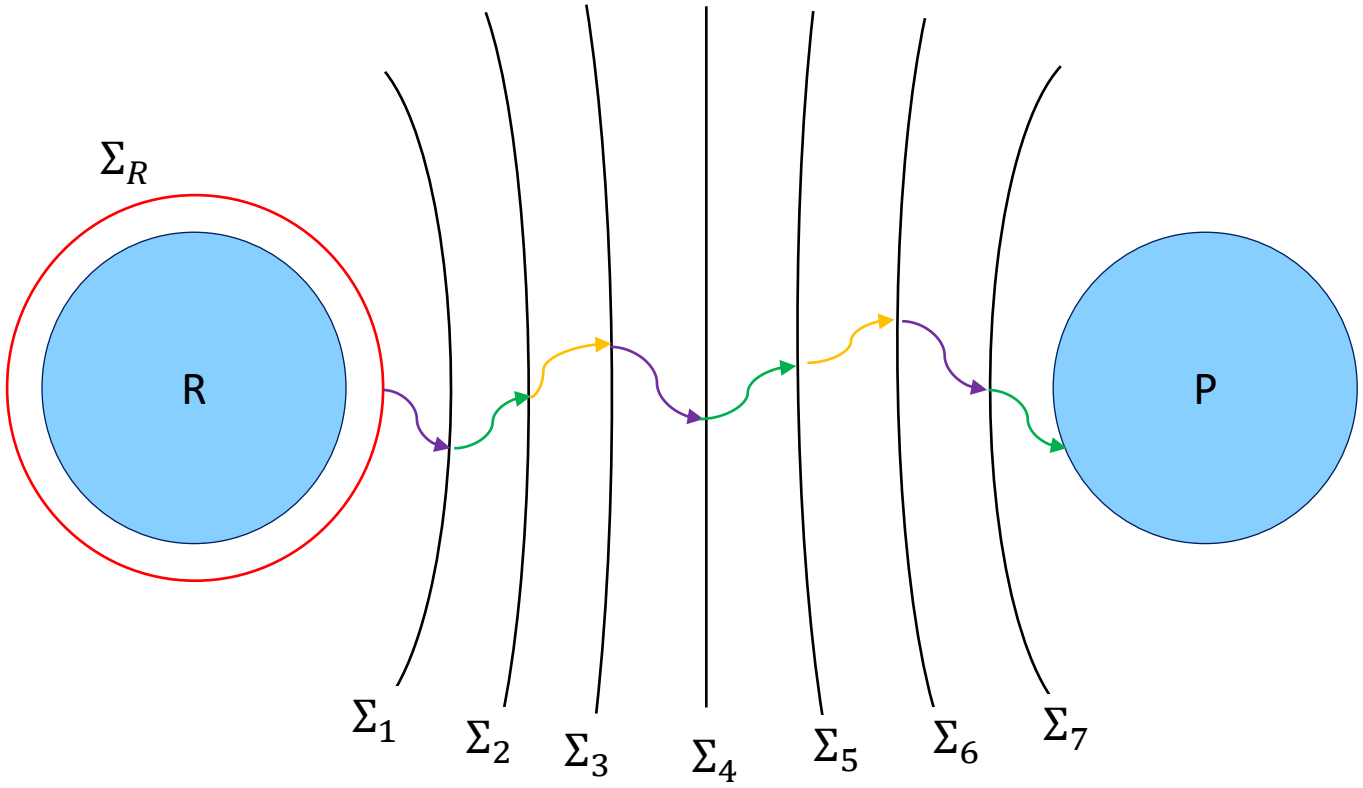
Not extremely sensitive to the definition of R and P

[1] Hänggi, P. Talkner, P. Borkovec, M. (1990) Reaction-rate theory: fifty years after Kramers *Reviews of Modern Physics* , Vol. 62, No. 2 American Physical Society (APS) p. 251-341

[2] Hill, T. (2012) Free energy transduction in biology: The steady-state kinetic and thermodynamic formalism. *Elsevier Science and Technology Books*

I. ADAPTIVE MULTI-LEVEL SPLITTING METHOD FOR REACTION RATES

What is a Multilevel Splitting estimator:



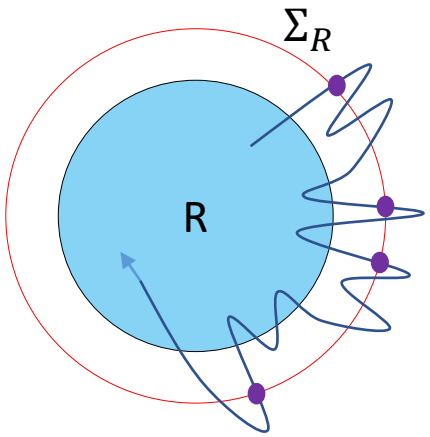
$$p_{R\to\Sigma_1}(\Sigma_R)p_{R\to\Sigma_2}(\Sigma_1)p_{R\to\Sigma_3}(\Sigma_2)p_{R\to\Sigma_4}(\Sigma_3)p_{R\to\Sigma_5}(\Sigma_4)p_{R\to\Sigma_6}(\Sigma_5)p_{R\to\Sigma_7}(\Sigma_6)p_{R\to P}(\Sigma_7) = p_{R\to P}\ (\Sigma_R)$$

How to place Σ_i and compute $p_{\mathrm{R}\to\Sigma_{i+1}}(\Sigma_i)$?

I. ADAPTIVE MULTI-LEVEL SPLITTING METHOD FOR REACTION RATES

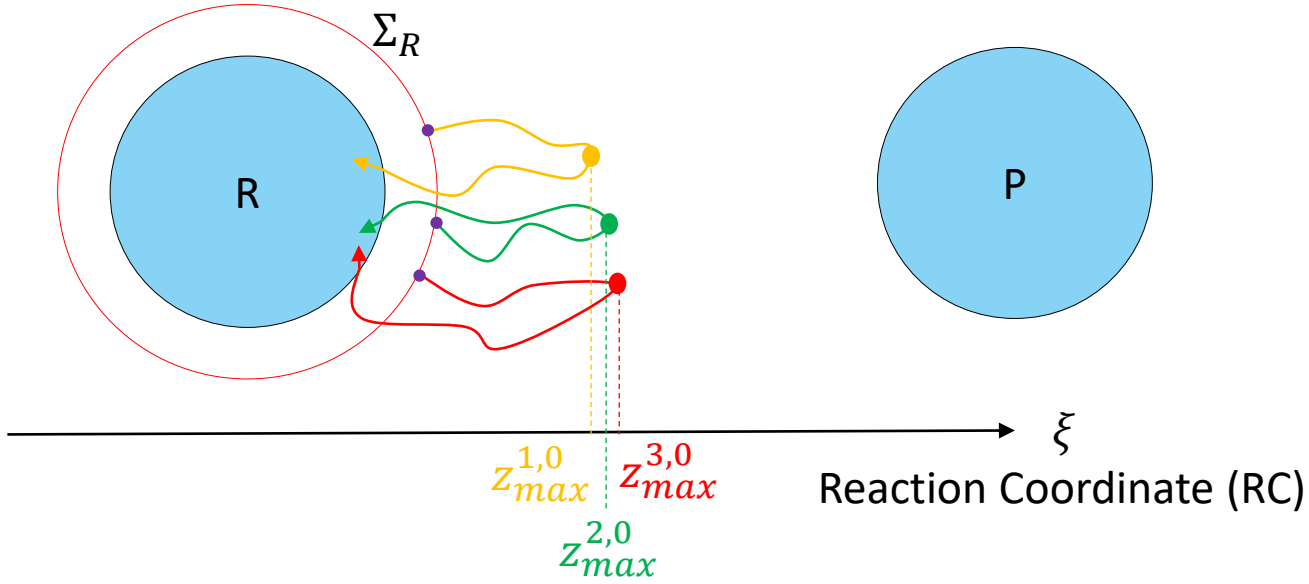
- AMS aims at estimating $p_{\Sigma \to P}$[1,2]. It can be split in 3 steps:
  1. Generating initial conditions on Σ and estimate $t_{R-\Sigma-R} = \frac{1}{\phi_R}$
  2. Initialize N replicas by running an unbiased dynamics until it reaches R or P. Set p = 1. Classify all the replicas by increasing ξ_{max}.
  3. Apply the AMS loop until all replicas have reached P.

1. Initial conditions and flux



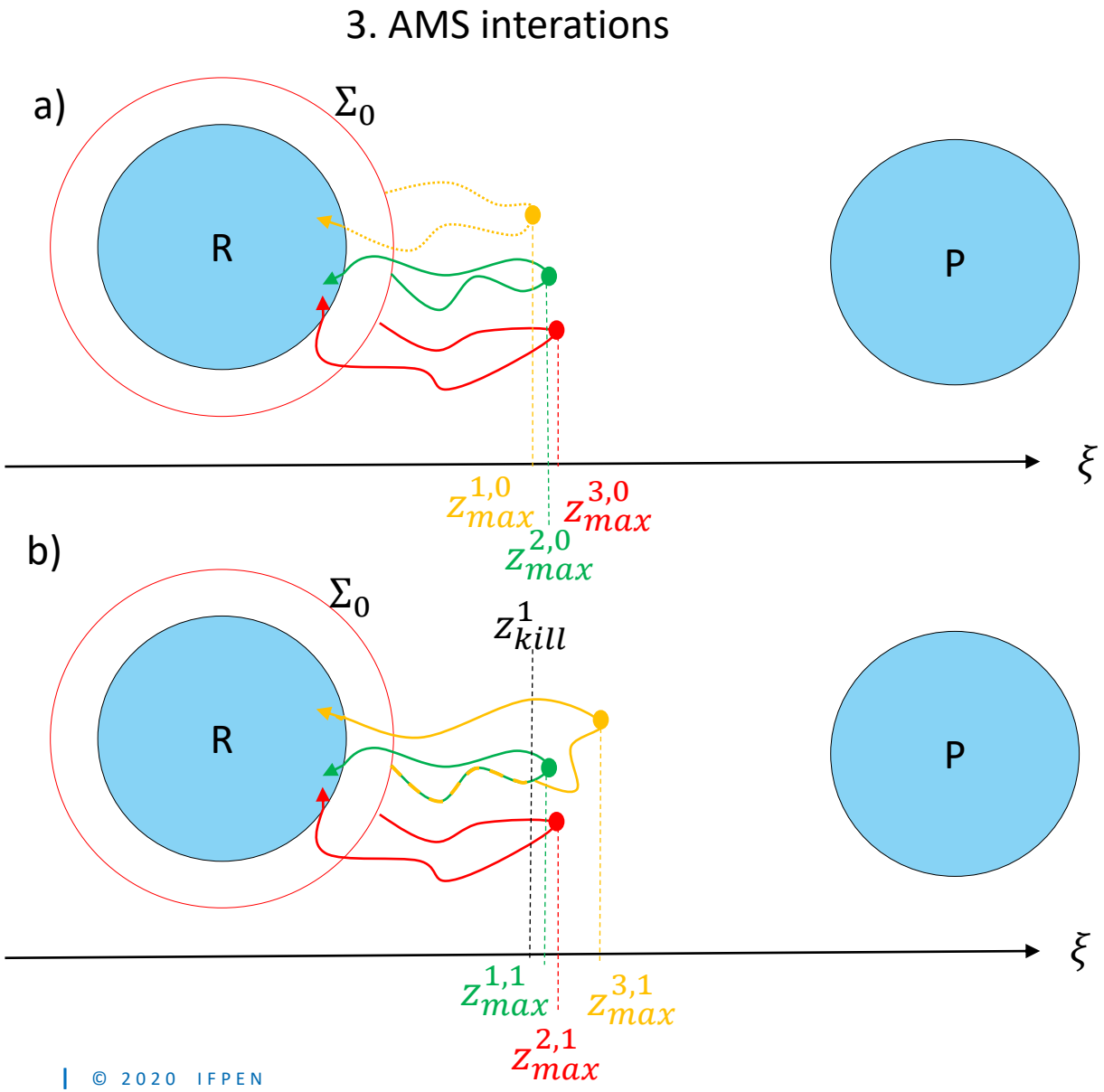
MD for Initial conditions.

2. Initialization



[1] F. Cérou, A. Guyader, ***Stochastic Analysis and Applications* 25**, 417-443 (2007).
[2] L. J. S. Lopes, T. Lelièvre, ***Journal of computational chemistry* 40**, 1198-1208 (2019).

I. ADAPTIVE MULTI-LEVEL SPLITTING METHOD FOR REACTION RATES



3. AMS iterations: $i \geq 0$

a) Save the smallest ($z_{max}^{1,i}$) as z_{kill}^{i+1} and delete all the trajectories that did not "go above" z_{kill}^{i+1}

b) Randomly select one trajectory within the remaining ones. Copy it until it reaches z_{kill}^{i+1} and continue it until it reaches R or P.

c) Classify all the replicas by increasing z_{max}.

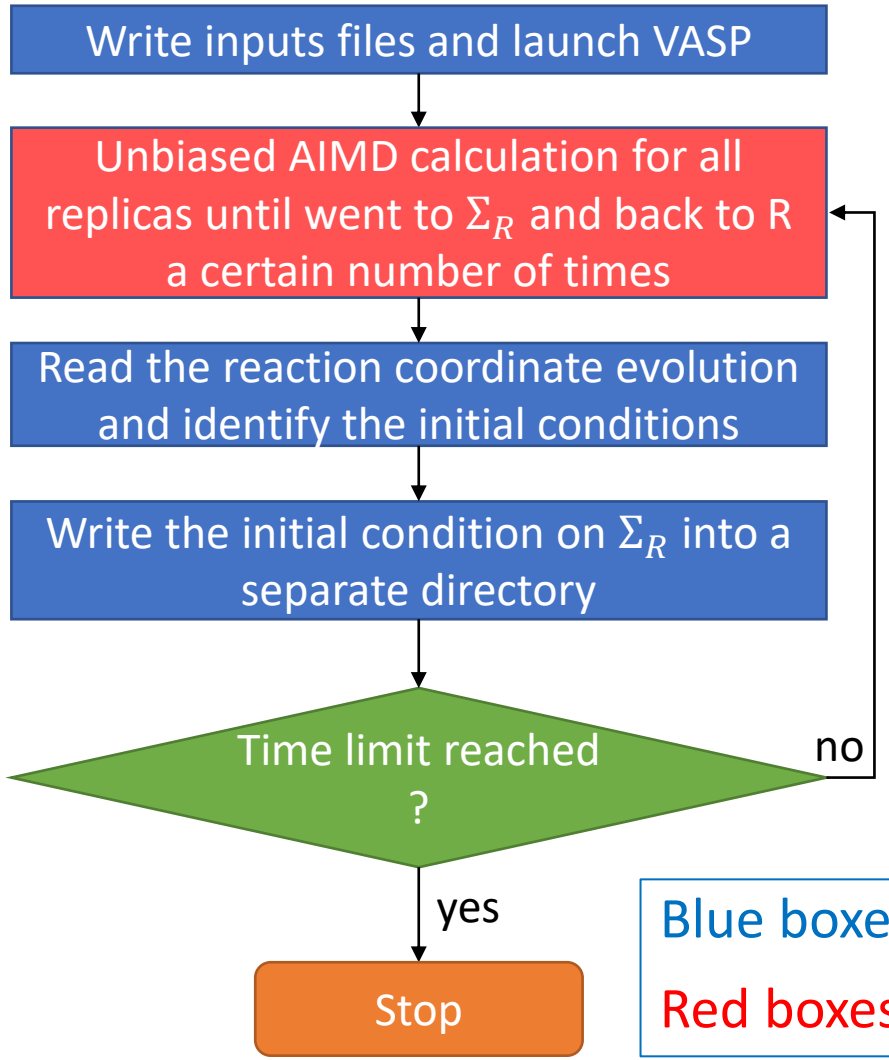
$$\tilde{p} = \prod_{i=0}^{i_{max}} \tilde{p}_{\Sigma_{z_{kill}^{i}} \to \Sigma_{z_{kill}^{i+1}}} = \left(1 - \frac{1}{N}\right)^{i_{max}}$$

Unbiased estimator: $\mathbb{E}[\tilde{p}] = p_{R-P}(\Sigma_R)$

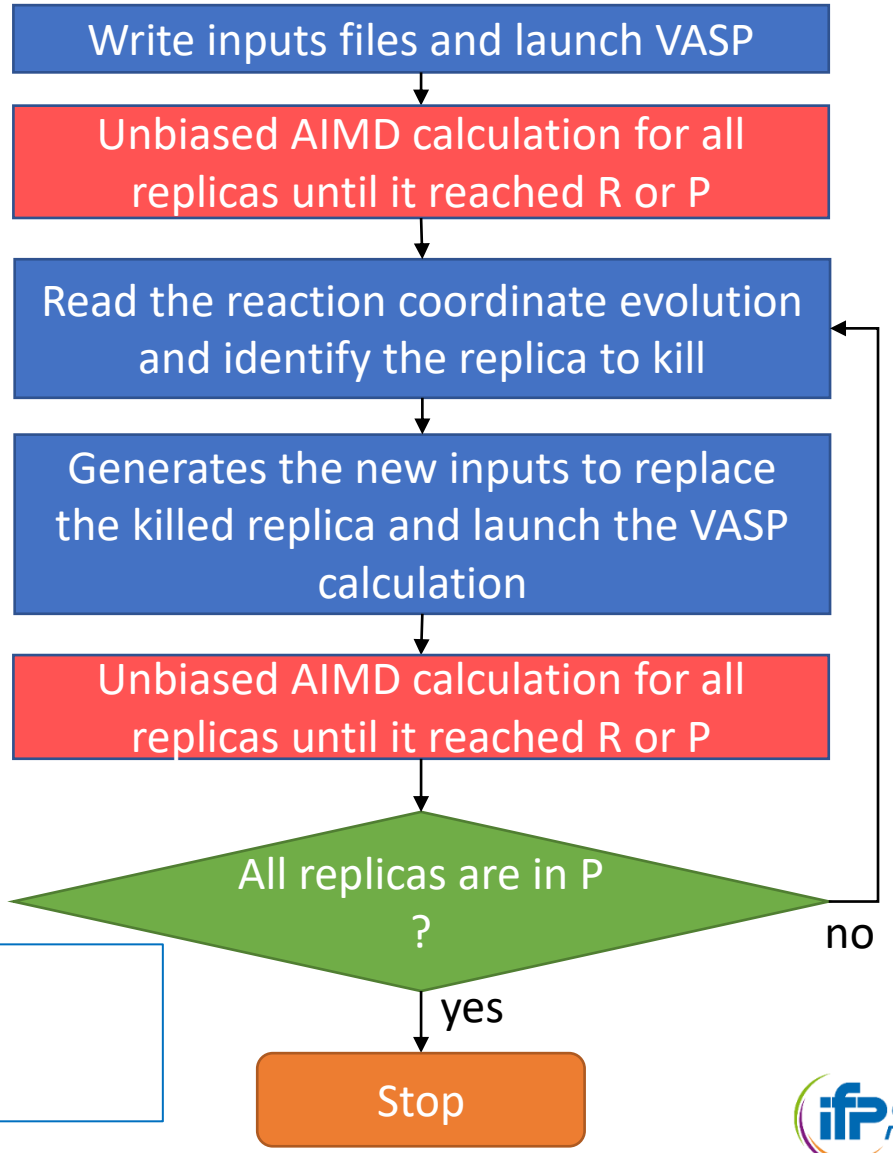
Variance depends on RC: $\mathrm{Var}[\tilde{p}] = f(\xi)$

I. ADAPTIVE MULTI-LEVEL SPLITTING METHOD FOR REACTION RATES

Sampling of initial conditions



Probability estimation with AMS



Blue boxes: python code

Red boxes: VASP code[1,2]

[1]Kresse, G.; Hafner, J. *Phys. Rev. B* **1993**, 47, 558–561.
[2]Kresse, G.; Joubert, D *Phys. Rev. B* **1999**, 59, 1758–1775.

II. AIMD METHOD APPLIED TO WATER DISSOCIATION ON (100) SURFACE

Multistate problem

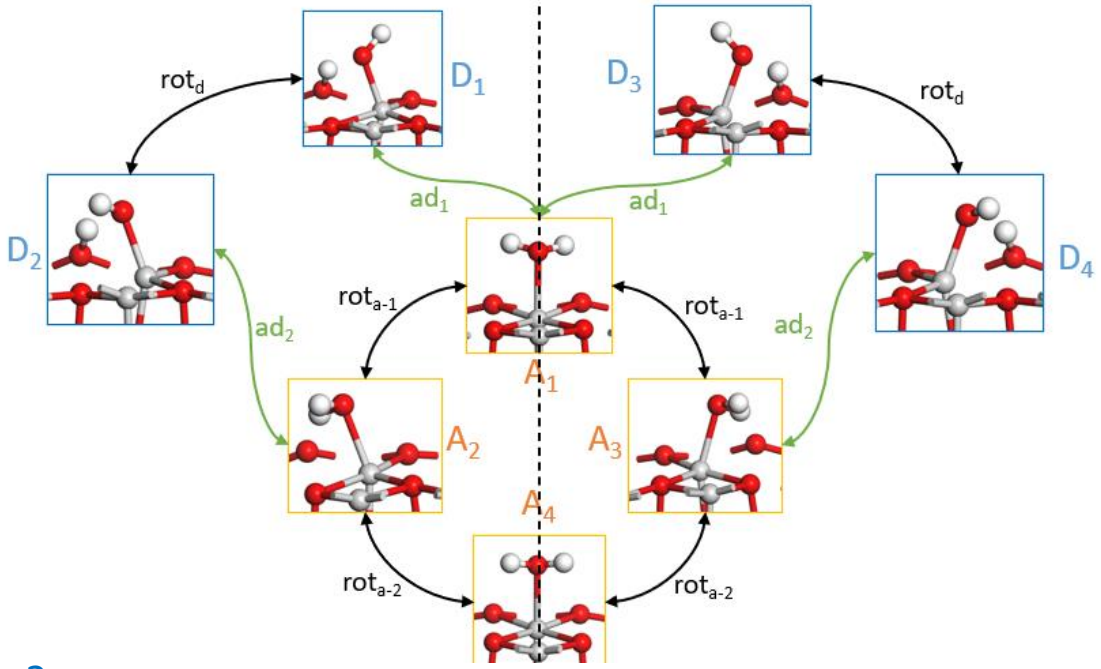
With
$R = A_1$
$\Sigma_R = \Sigma_{A_1}$
$P = A_2A_3 \cup A_4 \cup D_1D_3 \cup D_2D_4$

AMS can sample :

$A_1 \rightarrow A_2A_3$
$A_1 \rightarrow A_4$
$A_1 \rightarrow D_1D_3$
$A_1 \rightarrow D_2D_4$

→ Answers how A_1 can decompose ?

The most probable transition will be sampled, with precision conditioned by ξ



Metastable states of H_2O on the (100) surface of γ-alumina

With
$R = A_1 \cup A_2A_3 \cup A_4 \cup D_2D_4$
$\Sigma_R = \Sigma_{A_1}$
$P = D_1D_3$

AMS can sample :

$A_1 \rightarrow D_1D_3$

→ Focus specifically on one event

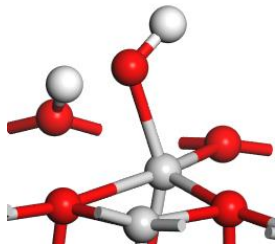
Quality of the sampling depends on ξ

II. AIMD METHOD APPLIED TO WATER DISSOCIATION ON (100) SURFACE

Method:

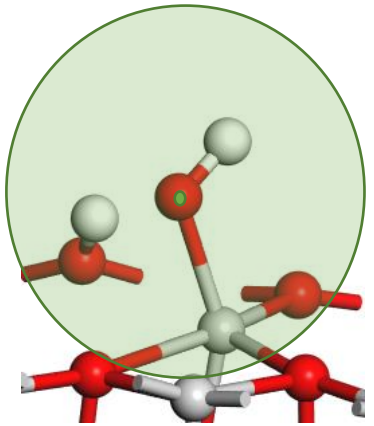
1. Identify the various metastable states (intermediates)
   → dissociated (D_i) or associated (A_i)
2. Run short dynamics in these states to sample Potential Energy Surface (PES) around the minima
3. SOAP[1] atom centered descriptors to numerically encode the structure for training the MLCV.

H
O
Al



Cartesian coordinates
(vector of 3N lines)

Select central atom



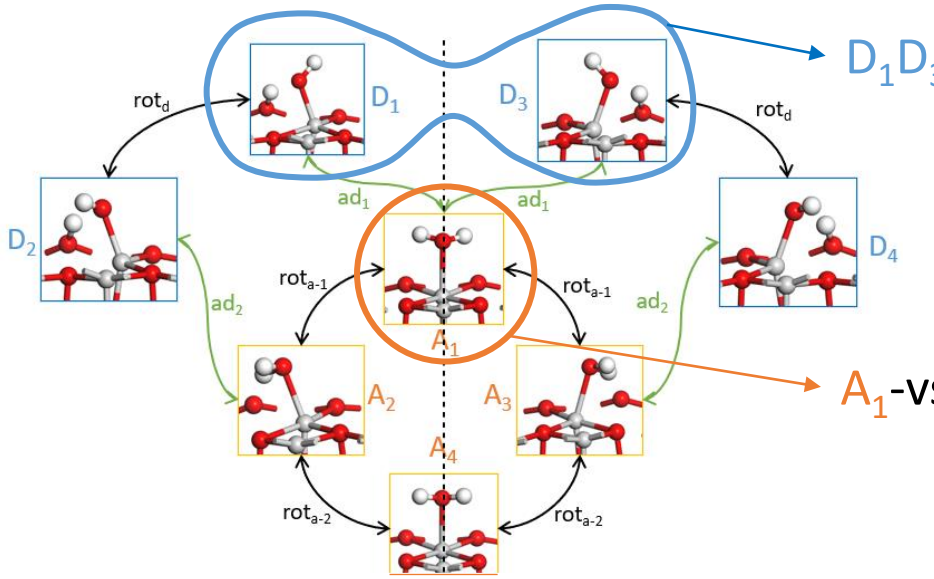
Atom centered descriptor of the structure
(vector of ~ 10^3-10^4 lines)

[1] Bartók, A. P., Kondor, R., & Csányi, G. (2013). On representing chemical environments. *Physical Review B*, *87*(18), 184115.

II. AIMD METHOD APPLIED TO WATER DISSOCIATION ON (100) SURFACE

SVM classifiers separate two sets of points by the highest margin plane.

SOAP-SVM CV : classifier decision function (f_X): algebraic distance to the plane.



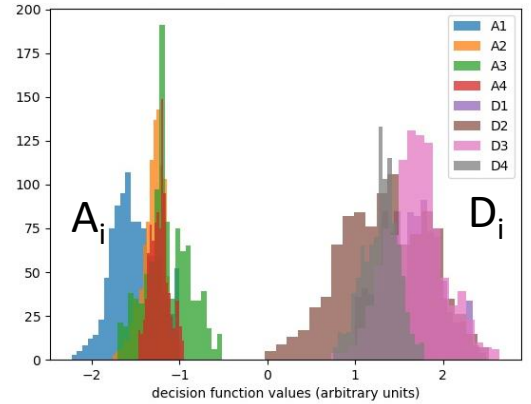
D$_1$D$_3$-vs-rest classifier

A$_1$-vs-rest classifier

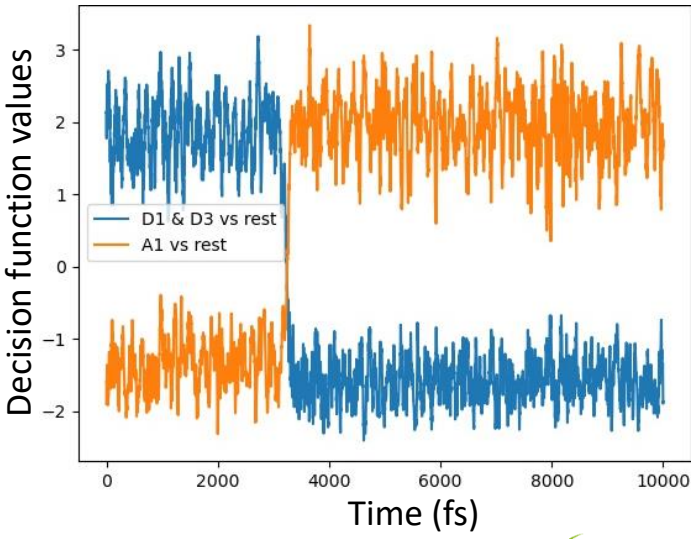
Classifier decision function interpretation:

$$f_X(\boldsymbol{q}) \in (-\infty, -1] \Leftrightarrow \boldsymbol{q} \in X$$

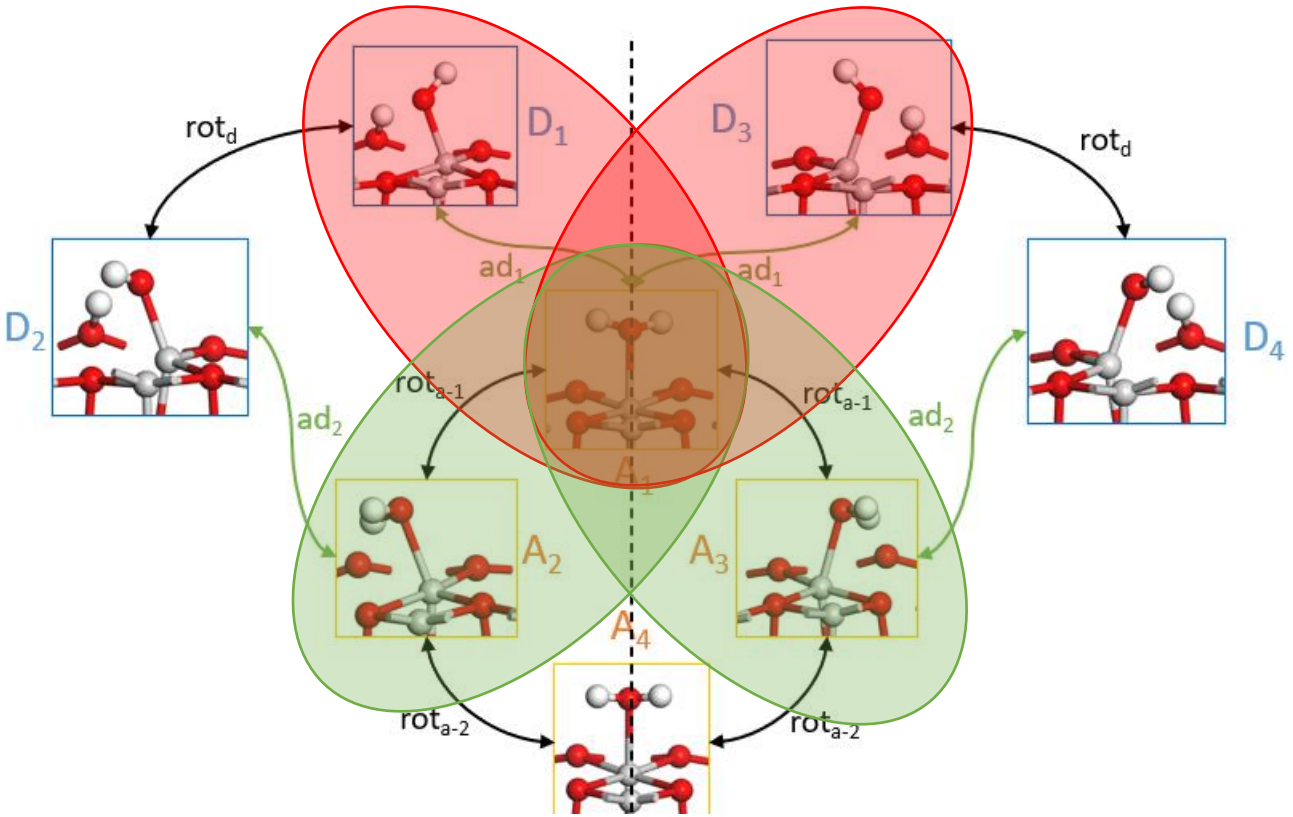
Histogram of SVM classifier decision function values



Training set : A1 = 0, D1 = 1



II. AIMD METHOD APPLIED TO WATER DISSOCIATION ON (100) SURFACE



Dissociation	Hill	hTST
$k_{A_1 \to D_1 D_3} =$	$1.6\ 10^{9}\ s^{-1}$	$3.4\ 10^{11}\ s^{-1}$
$k_{D_1 D_3 \to A_1} =$	$2.3\ 10^{10}\ s^{-1}$	$1.1\ 10^{12}\ s^{-1}$

Rotation	Hill	hTST
$k_{A_1 \to A_2 A_3} =$	$3.8\ 10^{10}\ s^{-1}$	$7.6\ 10^{10}\ s^{-1}$
$k_{A_2 A_3 \to A_1} =$	$1.5\ 10^{11}\ s^{-1}$	$2.1\ 10^{12}\ s^{-1}$

hTST rates are larger

Might come from entropy estimation.

~ 2 10^6 CPU Hours

Published

TP, G. Stoltz, M. Corral-Valero, A. Anciaux-Sedrakian, M. Moreaud, T. Lelièvre, P. Raybaud
J. Chem. Theory Comput. 2023, 19, 12, 3538−3550

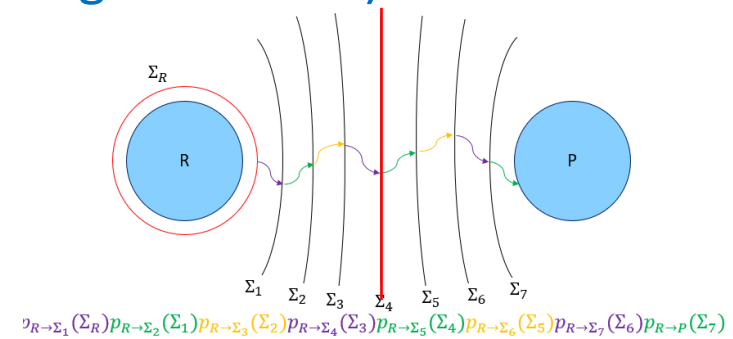
II. AIMD METHOD APPLIED TO WATER DISSOCIATION ON (100) SURFACE

Identify TS structures

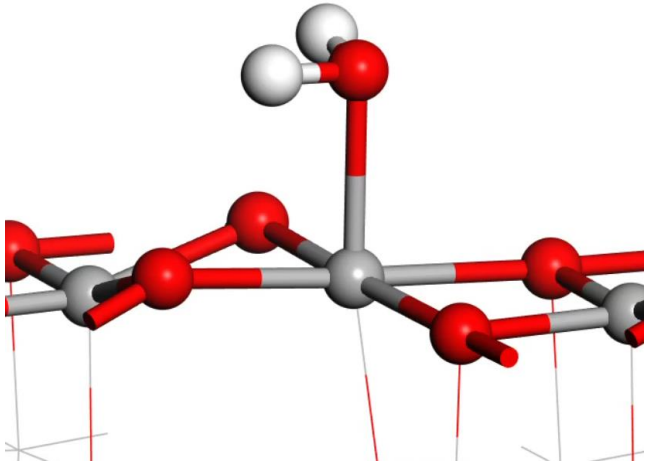
TS in the sense of committor function $p_{R\to P}$ (probability of reaching P before R)[1]

Find the level of the RC z^n_{kill} such that $p_{R\to P}\left(\Sigma_{z^n_{kill}}\right) = 0.5$

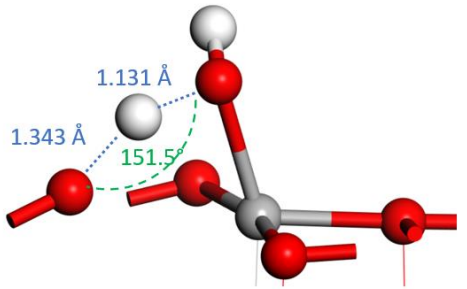
$$\prod_{i=n}^{i_{max}} \tilde{p}_{R\to\Sigma_{z^{i+1}_{kill}}}\left(\Sigma_{z^i_{kill}}\right) = 0.5$$



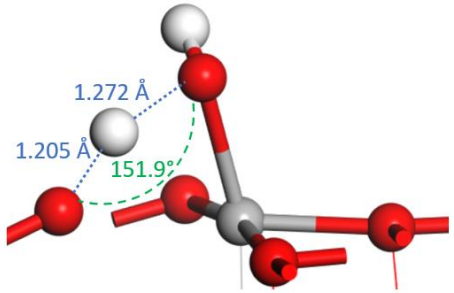
Along each trajectory, take the structure right after the level $\Sigma_{z^n_{kill}}$ is crossed, then find the average structure



Example for the $A_1 \to D_1D_3$ reaction



Saddle point



AMS estimated

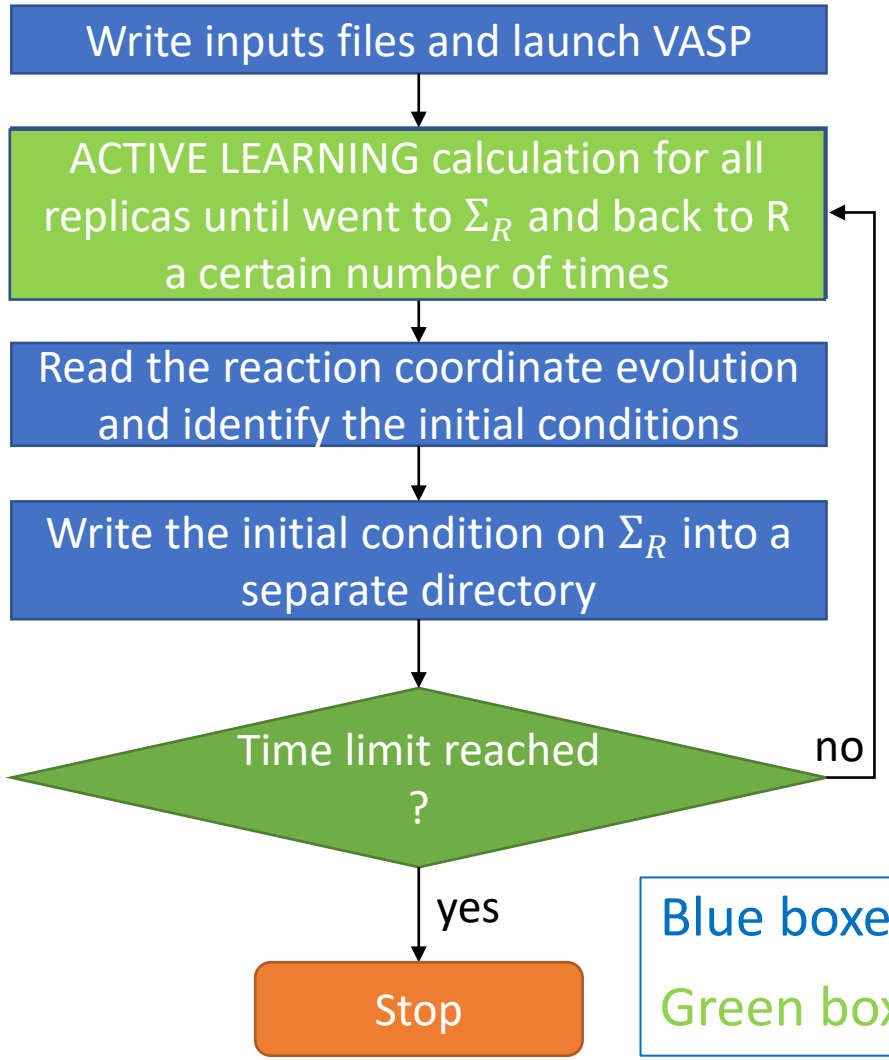
$p_{A_1\to D_1D_3} = 0.5$

[1] Vanden-Eijnden, E. Transition Path Theory (2006) in Computer Simulations in Condensed Matter Systems: From Materials to Chemical Biology Volume 1 Springer Berlin Heidelberg: Berlin, Heidelberg p. 453-493

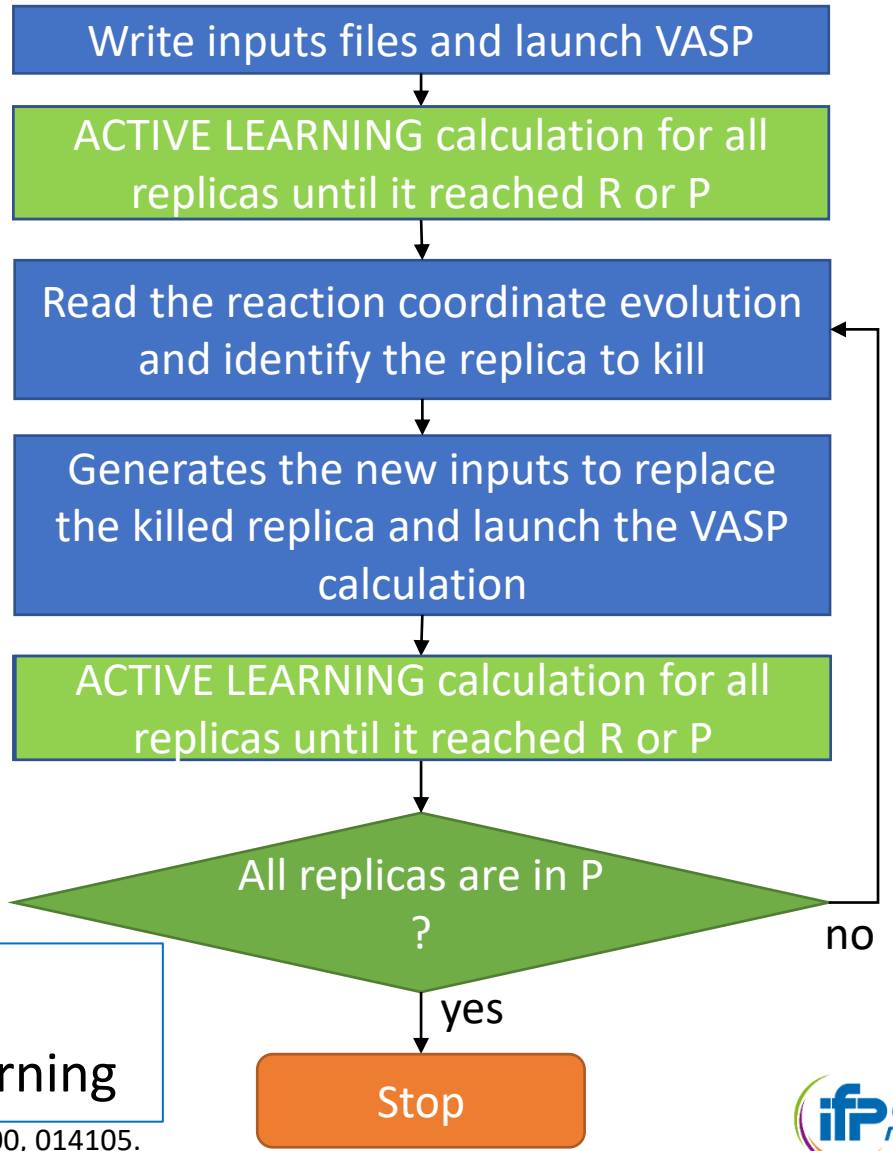
III. USING AMS WITH ACTIVE LEARNING

AMS IMPLEMENTATION WITH VASP (PLANE WAVE DFT)

Sampling of initial conditions



Probability estimation with AMS



Blue boxes: python code

Green boxes: VASP active learning

[1] Jinnouchi, R.; Karsai, F.; Kresse, G. *Phys. Rev. B* **2019,** 100, 014105.

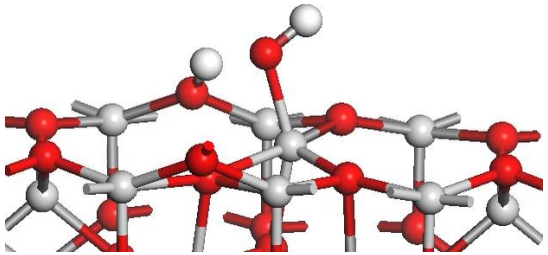
[2] Jinnouchi, R.; Miwa, K.; Karsai, F.; Kresse, G.; Asahi, R. *The Journal of Physical Chemistry Letters* **2020**, 11, 6946–6955.

III. USING AMS WITH ACTIVE LEARNING

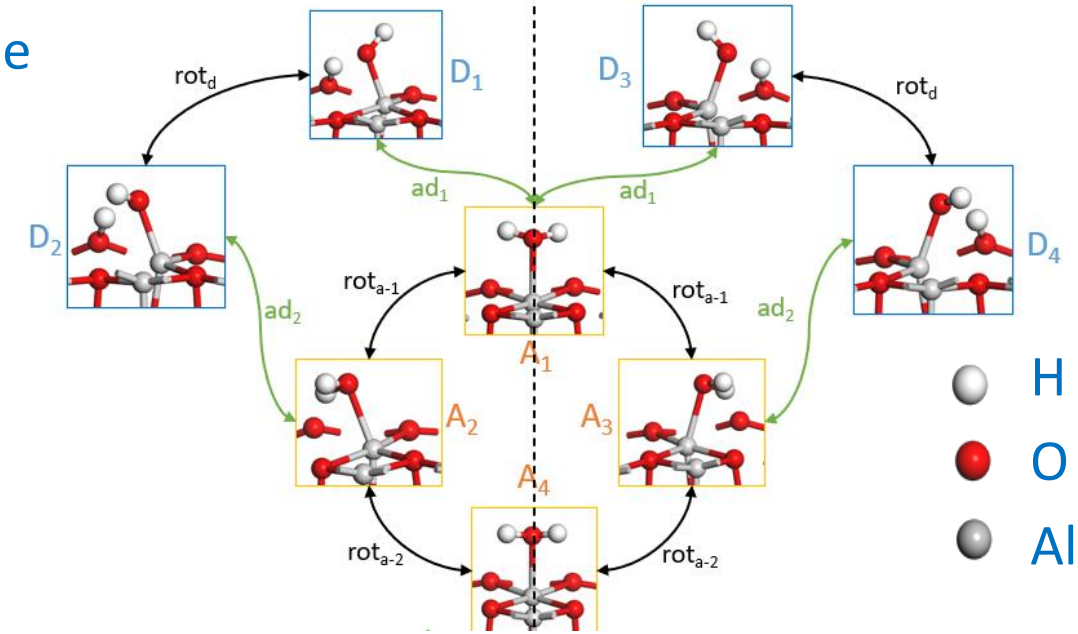
Method:

1. Identify the various metastable states (intermediates)
   → dissociated (D_i) or associated (A_i)
2. Run active learning dynamics in these states to sample Potential Energy Surface (PES) around the minima

D_1 dissociated structure



dt = 1 fs
Total time = 50 ps

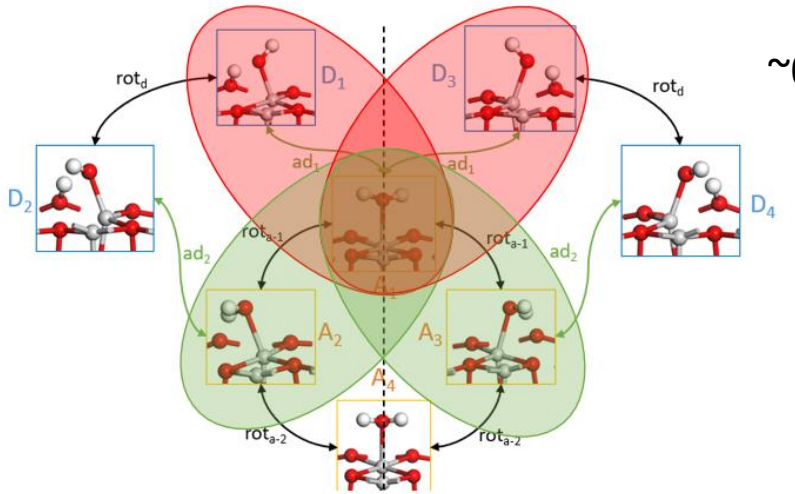


Identified structures and **intuitively** plausible transitions

3. Concatenate the dataset
   (containing E, Forces, and positions)

III. USING AMS WITH ACTIVE LEARNING

Wall clock time



~10 days

~0.5 days

~2 days

Dissociation with N_{rep} = 200 and M_{real} = 10

DFT

$k_{A_1 \to D_1 D_3} = 1.64\ 10^9\ s^{-1} \pm 1.59\ 10^9\ s^{-1}$

MLFF prediction

$k_{A_1 \to D_1 D_3} = 2.76\ 10^9\ s^{-1} \pm 3.81\ 10^9\ s^{-1}$

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with N_{rep} = 800 and M_{real} = 10

$k_{A_1 \to D_1 D_3} = 2.43\ 10^9\ s^{-1} \pm 1.15\ 10^9\ s^{-1}$

CONCLUSION AND PERSPECTIVES

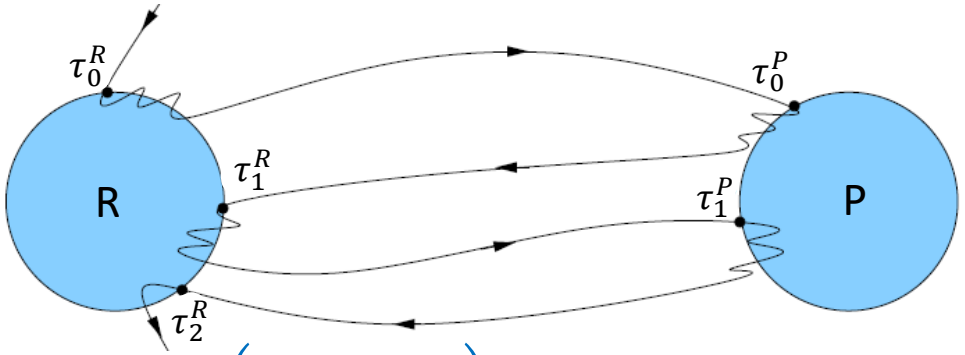
- hTST overestimate the DFT-MD rate estimated using AMS
- MLFF-MD and DFT-MD rates are consistent
- MLFF used in prediction mode drastically reduces de computational cost
- Current implementation of AMS with VASP limits the application of active learning
  → Restart does have an important cost for the active learning.
- Using FLARE[1] active learning with VASP as calculator of ab-initio seems a good opportunity as AMS could be implemented more easily
- Active learning of RC ξ can be included in the workflow

[1] Vandermause, J., Xie, Y., Lim, J.S., Owen, C.J. and Kozinsky, B., 2021. *Active learning of reactive Bayesian force fields: Application to heterogeneous hydrogen-platinum catalysis dynamics.*
arXiv preprint arXiv:2106.01949.

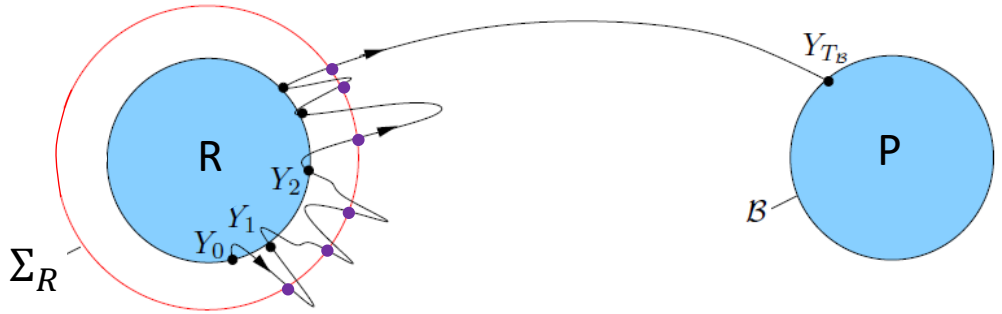
Thank you for you attention

I. ADAPTIVE MULTI-LEVEL SPLITTING METHOD FOR REACTION RATES

Transition time: $\frac{1}{k_{RP}} = t_{RP} = mean\left(\tau_i^R - \tau_i^P\right)$



We model the reaction time as: $\frac{1}{k_{RP}} = t_{RP} = \left(\frac{1}{p_{\Sigma_R - P}} - 1\right)(t_{R-\Sigma} + t_{\Sigma-R}) + t^{\dagger}_{R-\Sigma} + t_{\Sigma-P} \approx \frac{t_{R\to\Sigma\to R}}{p_{\Sigma_R\to P}} = \frac{1}{p_{\Sigma_R\to P}\,\phi_R}$

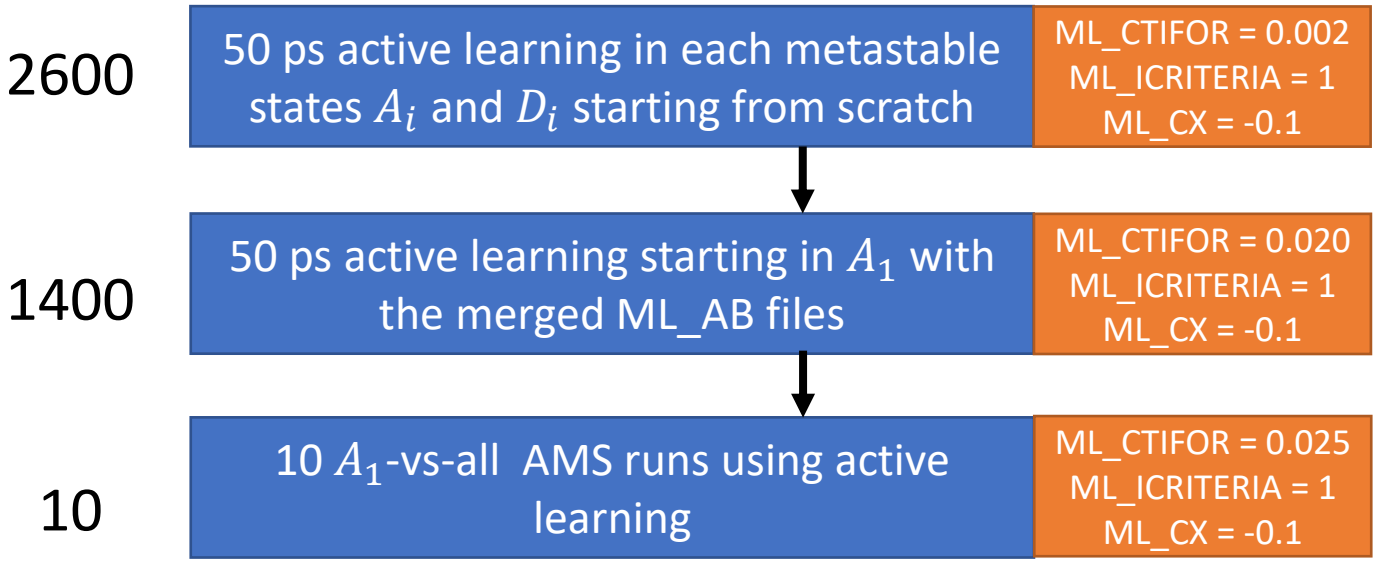


$p_{\Sigma_R - P}$: probability of reaching P before R when starting from Σ_R.

[1] Baudel, M., Guyader, A., & Lelièvre, T. (2020). On the Hill relation and the mean reaction time for metastable processes. *arXiv preprint, arXiv:2008.09790*.

III. USING AMS WITH VASP ACTIVE LEARNING

Number of structures added to the dataset:



III. USING AMS WITH ACTIVE LEARNING

Wall clock time	Number of structures		
~0.5 days	2600	50 ps active learning in each metastable states A_i and D_i starting from scratch	Final force mean Bayesian error 10 to 60 meV/Å
~0.5 days	1400	50 ps active learning starting in A_1 with the merged datasets	25 meV/Å
~10 days	10	10 A_1-vs-all AMS runs using active learning	25 meV/Å

Threshold updated using the stored Bayesian errors[1,2]

→ Transitions already sampled during first two steps?

[1] Jinnouchi, R.; Karsai, F.; Kresse, G. *Phys. Rev. B* **2019,** 100, 014105.
[2] Jinnouchi, R.; Miwa, K.; Karsai, F.; Kresse, G.; Asahi, R. *The Journal of Physical Chemistry Letters* **2020**, 11, 6946–6955.

II. AIMD METHOD APPLIED TO WATER DISSOCIATION ON (100) SURFACE

Use K-means clustering method to identify groups of trajectories.

Based on SOAP descriptor + PCA to describe 5 structures per trajectory.

5 Structures = First time trajectory cross RC iso-levels